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BRIEF COMMUNICATION

NUMERICAL SOLUTION OF THE GODDARD–HUANG EQUATIONS FOR THE SHAPE OF A FIBER IN FLOWING CONCENTRATED SUSPENSIONS

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The flow of concentrated suspensions of slender particles is encountered in a variety of practical applications such as reinforcement of plastics, the use of rodlike polymers in fiber spinning and papermaking, to name a few. The process of interest to us is papermaking, the current state of the art of which is based on ample dilution of the wood fiber suspensions in order to minimize fiber–fiber interactions. The use of dilute suspensions is, on the other hand, the reason for the excessively large size of papermaking equipment due to the large quantities of water that must be removed and handled. Forming paper from more concentrated suspensions has been considered as a possible avenue for reducing the size of papermaking equipment, and consequently, the capital intensiveness of the process.

Recently, we (Bonano 1984) pointed out that if future papermaking technology is to be based on using more concentrated fiber suspensions, we must be able to disrupt the fiber aggregates and maintain the fibers dispersed for sufficiently long periods of time in order to attempt their alignment. The results of that study showed that at high flow velocities, increasing the fiber concentration tends to retard the reformation of fiber aggregates once these are disrupted. Still to be established is the type of flow field needed to align the dispersed fibers and the time scales associated with their alignment. We address these issues in this communication by following the time evolving shape of a labeled fiber in a concentrated suspension subjected to simple shear and simple elongational deformations.

Goddard & Huang (1983) showed that for highly concentrated suspensions, the equations governing the shape of a flexible inextensible fiber in a homogeneous flow field given by Hinch (1976) simplify to

$$\frac{\partial \mathbf{x}}{\partial t} = K_t\left(\frac{\partial T}{\partial s}\right)\frac{\partial \mathbf{x}}{\partial s} + \mathbf{L}\cdot\mathbf{x} \qquad [1]$$

and

$$K_t\frac{\partial^2 T}{\partial s^2} = -\frac{\partial \mathbf{x}}{\partial s}\cdot\mathbf{L}\cdot\frac{\partial \mathbf{x}}{\partial s} \qquad [2]$$

where $\mathbf{x}(s, t)$ is the fiber position vector (assuming the slenderbody approximation applies), s is the distance along $\mathbf{x}$, t is time, $\mathbf{L}$ is the velocity gradient tensor, T is the tension along the fiber's axis and K_t is the longitudinal component of the mobility tensor of the fiber. Equations [1] and [2] are solved subject to the following initial and boundary conditions:

$$\mathbf{x}(0, s) = \mathbf{y}(s) \qquad [3]$$

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and

$$T(\pm l, t) = 0 \qquad [4]$$

where l is half the length of the fiber. Goddard & Huang obtained a closed-form analytical solution for these equations but it requires a complex transformation which must be inverted to obtain a picture of the actual fiber's shape. Thus, their approach to solving [1] and [2] is not amenable to observing the evolution in time of the shape of a fiber in different flow fields and as a function of initial shape and orientation. Equations [1] and [2] can be solved numerically in a straightforward fashion and this is the approach we adopted here.

Here, we solved [1]–[4] for two-dimensional systems where $\mathbf{L}$ is given by

$$\mathbf{L} = \begin{bmatrix} 0 & \gamma_s \\ 0 & 0 \end{bmatrix} \qquad [5]$$

for a simple shear flow, where γ_s is the shear rate, and by

$$\mathbf{L} = \begin{bmatrix} \gamma_E & 0 \\ 0 & -\gamma_E \end{bmatrix} \qquad [6]$$

for a simple elongational flow, where γ_E is the elongational rate. For both flow fields we tested the numerical scheme to insure that an initially straight fiber, which according to both Goddard & Huang (1983) and Hinch (1976) must behave like a rigid rod, remained undeformed.

RESULTS

Figures 1 and 2 show the shape of a fiber with different initial orientations with respect to the main flow direction as a function of time. Each figure is divided into two parts: Part a corresponding to the shear flow with $\tau = \gamma_s t$ and Part b to the elongational flow with $\tau^* = \gamma_E t$. The effects of flow field are readily discernible from these figures. The time to achieve alignment in a shear flow depends on the initial orientation of the fiber. Fibers which are initially oriented positively with respect to the flow (figure 1) require less time to orient themselves with the flow than those negatively oriented (figure 2). In figure 1, the fiber has straightened and is oriented at ~20° from the flow direction at $\tau = 3$, while in figure 2, it requires almost twice as long to achieve this shape and orientation. This is a result of the tumbling action of the fiber in a shear flow, a behavior observed by Jackson *et al.* (1982) in nondilute suspensions of rigid rods undergoing shear deformation.

The time required for a fiber to align in an elongational flow is practically independent

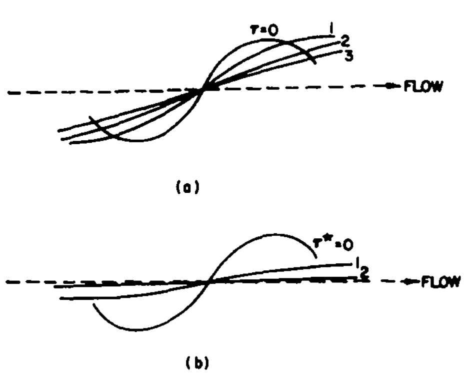

Figure 1. Evolving shape of initially positively oriented fiber in simple shear flow (Part a), and simple elongational flow (Part b).

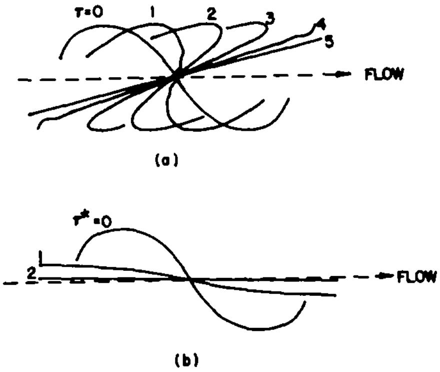

Figure 2. Evolving shape of initially negatively oriented fiber in simple shear flow (Part a) and simple elongational flow (Part b).

of initial orientation; in both figures, we observe that by $\tau^* = 2$, the fibers are essentially straight and aligned with the flow. Furthermore, the shapes of the fibers at $\tau^* = 1$ and 2 indicate that fibers in an elongational flow tend to reorient their axis with the flow before straightening by being pushed toward the centerline. This action seems to be almost independent of the initial orientation. This observation may prove to be important from a papermaking standpoint since, fiber–fiber interactions which were not taken into account here, may actually prevent the fibers from completely straightening, yet they may still be able to align with the flow. In papermaking, the concern is not so much with the actual shape of the fibers as with their orientation.

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